

Name: Solutions  
Date: 4/24/2025

Math 130  
Quiz 15

Some formulas you may need:  $\chi^2 = \frac{(n-1)s^2}{\sigma^2}$   $df = n-1$

1. (10 points) The Coca-Cola bottling company has developed a new machine that they claim is much more consistent when it fills cans of Coke. The standard deviation for the amount of Coke filled in their cans by their older machines is estimated to be about 0.15 ounces. To determine if this new machine is better, the new machine was used to fill 100 cans of Coke. The standard deviation of the amount of Coke in the cans from this sample is 0.12 ounces. Use a 0.05 significance level to test the claim that the new machine is more consistent than the old machine when it fills cans of Coke.

### Hyp. Test

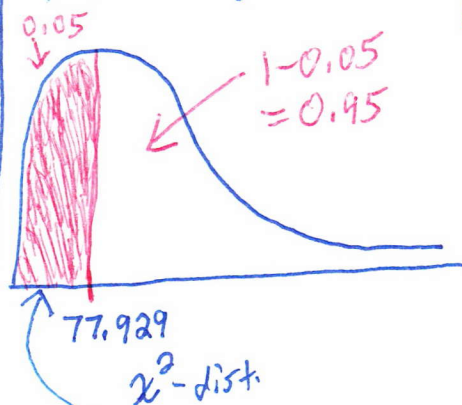
$H_0: \sigma = 0.15$  ounces

$H_1: \sigma < 0.15$  ounces

$\sigma$  = The standard deviation of the number of ounces of soda in all cans filled by the new machine

### Rejection Region

$\alpha = 0.05$   $df = n-1 = 100-1 = 99$  (use 100)



### Test stat

$n = 100$   $s = 0.12$

$$\begin{aligned}\chi^2 &= \frac{(n-1)s^2}{\sigma^2} \\ &= \frac{(100-1)(0.12)^2}{(0.15)^2} \\ &= \boxed{63.36}\end{aligned}$$

### p-value

$\alpha = 0.05$   $df = 99$



$$\begin{aligned}p\text{-value} &= P(\chi^2 < 63.36) \\ &= 0.0020282566\end{aligned}$$

Is  $p\text{-value} < \alpha$ ?

$$0.0020282566 < 0.05?$$

Yes!

### Conclusion

Reject  $H_0$ !

Evidence suggests that the new machine is more consistent than the old machine when it fills cans of Coke.

### Extra Credit

1. (1, 2, 2 points) Suppose you are performing the following hypothesis test:

$$H_0: \sigma = 18$$

$$H_1: \sigma \neq 18$$

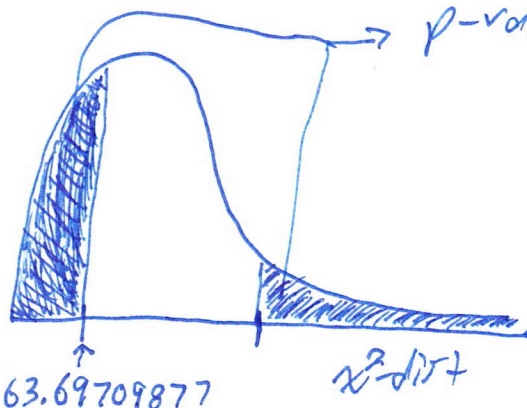
You take a sample of size 75, and calculate a sample standard deviation of 16.7.

- a) Calculate the test statistic

$$n = 75$$
$$s = 16.7$$
$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(75-1)(16.7)^2}{(18)^2} = \boxed{63.69709877}$$

- b) Find the P-value of the test statistic

$$\alpha = ?$$
$$df = n - 1$$
$$= 75 - 1$$
$$= 74$$
$$p\text{-value} = 2 \cdot P(\chi^2 < 63.69709877)$$
$$= \boxed{0.4041473716}$$



bec.  $s < \sigma$  ( $16.7 < 18$ )

- c) In this situation would you reject  $H_0$ ? Why or why not?

$$p\text{-value} = 0.4041473716$$

is big

$\Rightarrow$  Do not reject  $H_0$ !

2. (3 points) What is the meaning of the significance level when performing a hypothesis test at the 0.07 significance level?

If you perform the same hypothesis test many times, each time with a new sample, you will reject  $H_0$  when  $H_0$  is true about 7% of the time.

3. (2 points) Describe what a type II error is.

when you do not reject  $H_0$  when  $H_0$  is false.